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Non-statistical fluctuations in ^{16}O-AgBr interaction at 60A GeV in terms of factorial correlators

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Abstract. An analysis of ^{16}O-AgBr interaction data at 60A GeV in terms of factorial correlators is presented. The correlated moments are found to increase with decreasing bin-bin separation D, following a power law within the region $D \leq 1$. The data are also consistent with the dimension-independent scaling relation proposed by Seixas.

PACS. 24.60.Ky Fluctuation phenomena

Recently, much attention has centered around the study of non-statistical fluctuations in pionisation at high and ultra-high energy nuclear collisions. The concept of intermittency was introduced in the field of multi-particle production by Bialas and Peschanski [1], from the theory of turbulence where it is used to measure the effects of bursts in a turbulent system. Intermittency is signaled by the power law behaviour of the scaled factorial moments with increasing spatial resolution of the particle detection procedure. The unique feature of this moment method is that it can detect and characterize the dynamical density fluctuations in particle spectra eliminating the statistical fluctuation which is always present in events with finite multiplicity [1]. The strength of the fluctuation is characterized by the intermittency exponent ϕ which may be obtained from the relation: (factorial moment) $\propto$ (phase-space size)$^{-\phi}$. In the field of "intermittency" study analysis of various experimental data [2–11] has been performed using the Scaled Factorial Moments (SFMs) [1] as the tool in different phase space variables. Such tremendous enthusiasm in studying intermittency effect stems from one major curiosity about the possible formation of quark gluon plasma (QGP) in nuclear collision. But later on it is found that different data support different interpretations. So a more discriminative information is needed experimentally.

It may be mentioned here that the other possible approach could be the test for intermittency in terms of Factorial Correlators (FCs), as suggested by Bialas and Peschanski [1]. Further the FCs not only measure the non-statistical fluctuation but also correlate the fluctuations in different regions of phase space, providing additional information. However, analysis of data in terms of FCs is relatively scarce. This paper analyses ^{16}O-AgBr interaction data at 60A GeV in the framework of the correlated factorial moments in pseudorapidity phase space using nuclear emulsion technique. It would be rather interesting to see what the data reveal in terms of FCs, confronting in particular the most discussed α-model which is described below:

α-model [1] describes each multiparticle event as a series of cascading steps starting from some initial phase space interval Δ and dividing it into smaller ones. In a single step, s, each of the current intervals of size $\delta^{(s)}$ is divided into λ smaller ones $\delta^{(s+1)} = \delta^{(s)}/\lambda$. The particle density in a bin at step, s, is obtained by multiplying the corresponding density in the step $(s-1)$ by a particular value of random "enhancement/suppression" variable W with the probability distribution $r(W)$ obeying constraints

$$\langle W \rangle = \int r(W) W \mathrm{d}W = 1 \qquad (1)$$

W is generated independently for each step and for each branch. The highest resolution δ is achieved after n steps, where $\Delta/\delta = \lambda^n = M$ is the total number of bins into which the whole interval Δ has been divided. The moments of the density ρ_m in the m-th bin have a power law dependence on the resolution δ:

$\langle \rho_m^q \rangle \sim (\Delta/\delta)^{\phi q}$ where ϕ_q are the intermittency exponents:

$$\phi_q = \ln\langle W^q \rangle / \ln \lambda,$$

Now, the correlated moments or the FCs involve more than one part or interval at a given step, providing correlations between the intervals. These FCs can also be expressed as a function of the random W's. The α-model predicts for FCs (i) a power law increase with decreasing distance between the intervals and (ii) the independence of FCs with the size of the intervals. The nuclear emulsion provides a rather convenient method of studying a high energy interaction for its (i) efficiency as a detector,

(ii) high spatial resolution and (iii) 4π geometry coverage. It thus helps in studying fluctuations even in very small pseudorapidity intervals, detecting all the produced charged particles with such fine resolution.

A stack of G5 nuclear emulsion plates horizontally exposed to an ^{16}O beam, having an average beam energy of 60 GeV per nucleon at CERN SPS has been used in this work. Leitz metalloplan microscopes provided with semi-automatic scanning stage are used to scan the plates, using objectives 10× in conjunction with a 10× ocular lens. The scanning is done by independent observers to increase the scanning efficiency which turns out to be 98%. Criteria to select the events are:

(a) The beam track did not exceed 3° from the mean beam direction in the pellicle.
(b) Events having interactions within 20 μm from the top or bottom surface of the pellicle are rejected.
(c) The incident beam tracks are followed in the backward direction to ensure that selected events did not include interactions from the secondary tracks of other interactions, the latter events are removed from the sample.

Following the above selection procedure we have chosen 250 primary events of ^{16}O-AgBr interactions at 60A GeV. All the tracks are classified as usual [12]:

(i) The target fragments with ionization $> 1.4I_0$ (I_0 is the plateau ionization) produce either black or grey tracks. The black tracks with range <3 mm represent target evaporation particles (the light nuclei evaporated from the target) of $\beta < 0.3$, singly or multiply charged particles.

(ii) The grey tracks with a range ≥3 mm and having velocity $0.7 \geq \beta \geq 0.3$ are mainly images of fast target recoil protons of the energy range up to 400 MeV.

(iii) The relativistic shower tracks with ionization $< 1.4I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle. They are believed to carry important information about the nuclear reaction dynamics.

(iv) The projectile fragments formed a different class of tracks with constant ionization, long range and small emission angle.

To ensure the target in the emulsion to be Ag/Br, only such events are chosen in which the number of heavily ionizing tracks exceed eight. The heavily ionizing particles of types (i) and (ii) belong to the target nucleus and those of type (iv) to the projectile nucleus. Particles of type (iii) are produced in the final state of the interaction. To distinguish between the single charged produced particles (pions) and projectile carrying same charge the following procedure has been adopted: the projectile fragments are expected to fall inside a cone of semivertical angle θ_c [13] with the direction of the projectile, where θ_c is equal to $0.2/p_{\mathrm{beam}}$, p_{beam} (GeV/c) is the incident beam momentum per nucleon, we checked emission angle of all shower tracks and excluded those with emission angle less than θ_c. In this way the singly charged projectile fragments were eliminated from the sample. The spatial angle of emission (θ), in the laboratory frame, of all the produced particles, is measured by taking the space co-ordinates (x, y, z) of a point on the track, another point on the incident beam and the production point by using oil immersion objectives (100× in conjunction with a 10× ocular lens). The variable pseudorapidity is obtained from the emission angle (θ) by the relation, $\eta = -\ln(\tan\theta/2)$.

In terms of the experimental parameters, the SFMs which measure local density fluctuations in pseudorapidity phase space, are defined as

$$\langle F_q \rangle = M^{q-1} \sum_{j=1}^{M} \langle n_j(n_j-1)\ldots(n_j-q+1)\rangle / \langle n_j \rangle^q, \quad (2)$$

where M is the number of bins into which the total pseudorapidity range $\Delta\eta$ has been divided. $\delta\eta$ is the width of each pseudorapidity bin. n_j is the multplicity in the j-th bin. $\langle\ \rangle$ denotes the average over all events and q the order of moments. The presence of intermittency gives the power law,

$$\langle F_q \rangle \propto (\Delta\eta/\delta\eta)^{\varphi q}, \quad (3)$$

where φ_q is defined as the "intermittency exponent" of the q-th order, and F_q is a measure of local density fluctuations in pseudo rapidity phase space.

FCs on the other hand, describe the correlation among local fluctuation in different regions of phase space and is defined as [14]

$$F_{ij}^{k,l}(\delta\eta) = \frac{\langle n_k(n_k-1)\ldots(n_k-i+1)n_l(n_l-1)\ldots(n_l-j+1)\rangle}{\langle n_k(n_k-1)\ldots(n_k-i+1)\rangle\langle n_l(n_l-1)\ldots(n_l-j+1)\rangle} \quad (4)$$

where n_k and n_l are the multiplicity in k-th and i-th bins respectively. The factorial correlators are calculated for each combination of the k and l for a particular $\delta\eta$ and averaged for all possible bin-bin combinations with a given separation $D = d\delta\eta$, where $d = |k-l|$, of the k-th and l-th bins. We have however calculated the average for all bin combinations with a given distance D, to get

$$\begin{aligned} C_{ij}(D,\delta\eta) &= 1/[2(M-d)]\left[\sum_{k=1}^{M-d} F_{ij}^{k,k+d}(\delta\eta) + \sum_{l=1}^{M-d} F_{ij}^{l+d,l}(\delta\eta)\right] \\ &= 1/[2(M-d)]\sum_{k=1}^{M-d}\left[F_{ij}^{k,k+d}(\delta\eta) + F_{ji}^{k,k+d}(\delta\eta)\right] \end{aligned} \quad (5)$$

where $F_{ij}(k,l,\delta\eta) = F_{ji}(l,k,\delta\eta)$. The correlators, C_{ij}, can now be studied as a function of bin-bin separation D and bin with $\delta\eta$. According to the α-model [1], the intermittent behaviour of pionization implies that $\langle C_{ij}\rangle$ would depend on the bin-bin separation D, following a power law

$$C_{ij} \propto D^{-\phi_{ij}} \quad (6)$$

and that it should be independent of $\delta\eta$. However, it has been shown in [14] that this $\delta\eta$-independence of FCs is a common feature of any model dealing with short range correlation functions.

Table 1. The slope values of best fits from the plot of $\ln C_{ij}$ *versus* $-\ln D$ in the region $D \leq 1$ for different bin widths

Order of the correlators	Slope (ϕ_{ij}) for $D \leq 1$			
ij	$\delta\eta = 0.1$ ($0.1 \leq D \leq 1.0$)	$\delta\eta = 0.13$ ($0.13 \leq D \leq 0.93$)	$\delta\eta = 0.2$ ($0.2 \leq D \leq 1.0$)	$\delta\eta = 0.4$ ($0.4 \leq D \leq 0.8$)
11	0.218 ± 0.018	0.324 ± 0.008	0.342 ± 0.034	0.388 ± 0.004
21	0.476 ± 0.008	0.511 ± 0.019	0.527 ± 0.021	0.545 ± 0.005
31	0.560 ± 0.015	0.574 ± 0.035	0.640 ± 0.009	0.662 ± 0.007
22	0.754 ± 0.028	0.766 ± 0.035	0.847 ± 0.022	0.944 ± 0.008
32	0.898 ± 0.036	0.905 ± 0.056	1.019 ± 0.046	1.053 ± 0.009
33	1.061 ± 0.039	1.081 ± 0.066	1.220 ± 0.086	1.274 ± 0.011

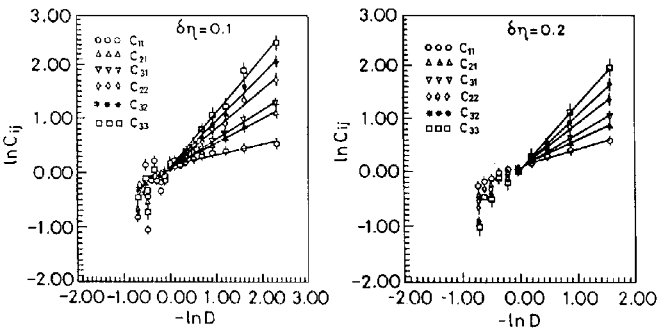

Fig. 1. The variation of $\ln C_{ij}$ with $-\ln D$ for (a) $\delta\eta = 0.1$ and (b) $\delta\eta = 0.2$

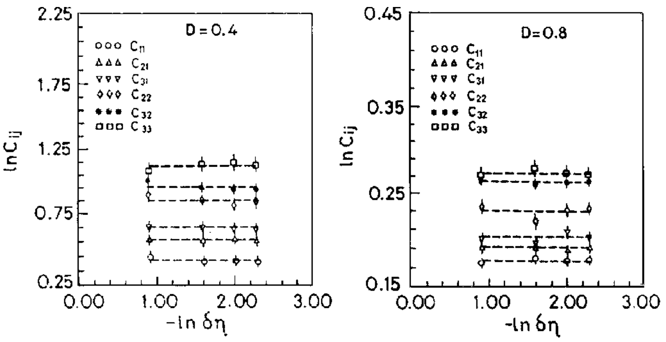

Fig. 2. Log – log plot of factorial correlators C_{ij} *versus* inverse bin widths ($1/\delta\eta$) revealing the independence of C_{ij} on the bin size for (a) $D = 0.4$ and (b) $D = 0.8$

We have performed our analysis in the pseudorapidity space of width $\Delta\eta = 4.0$ around the peak of the distribution. The power law (6) dependence has been studied for four different values of the bin size ($\delta\eta = \Delta\eta/M = 0.4$, 0.2, 0.13 and 0.1) by choosing $M = 10$, 20, 30 and 40 respectively. The variation of $\ln C_{ij}$ as a function of $-\ln D$ has been shown in Fig. 1(a–b) for $\delta\eta = 0.1$ and 0.2. The error bars indicate the standard statistical errors of the F_{ij} values. For clarity, error bars are shown at few points. In each case, $\ln C_{ij}$ is found to increase with $-\ln D$ and the validity of the power law (6) is also observed. The plots exhibit that they are linear only in restricted regions of D-values namely, $D \leq 1.0$. Similar behaviour have also been observed for $\delta\eta = 0.13$ and 0.4. So, linear fits have been performed only for selected ragions of D values. The selected D ranges are $0.4 \leq D \leq 0.8$, $0.2 \leq D \leq 1.0$, $0.13 \leq D \leq 0.93$, and $0.1 \leq D \leq 1.0$ for bin widths 0.4, 0.2, 0.13, and 0.1, respectively. Table 1 shows a systematic decrease in the slopes with decreasing bin size for any particular order ($i \times j$). A rise in the slope values for increasing order of moments for a given $\delta\eta$ is also evident.

In Fig. 2(a), the plot of $\ln C_{ij}$ *vs.* $-\ln \delta\eta$ for a fixed D value (0.4) of our data, reveals the independence of C_{ij} on the bin size $\delta\eta$, as expected from the α-model. The horizontal dashed lines are fits through the average values to facilitate observation. The independence of C_{ij} on the bin size $\delta\eta$ has also been tested for $D = 0.8$ and is shown in Fig. 2(b). We thus observe that the features of the α-model – the $\delta\eta$-independence and power like D-

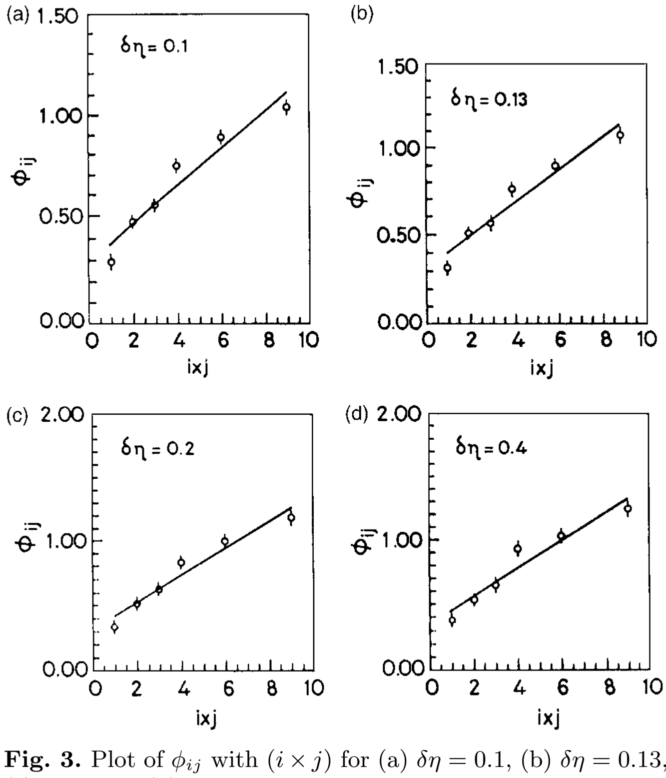

Fig. 3. Plot of ϕ_{ij} with ($i \times j$) for (a) $\delta\eta = 0.1$, (b) $\delta\eta = 0.13$, (c) $\delta\eta = 0.2$, (d) $\delta\eta = 0.4$

Table 2. Comparison of ϕ_2 with slope values of ϕ_{ij} *versus* $(i \times j)$ plots for different pseudorapidity bin widths

$\delta\eta$	$\Delta\phi_{ij}/\Delta(i \times j)$	$\langle \Delta\phi_{ij}/\Delta(i \times j) \rangle$	ϕ_2
0.4	0.111 ± 0.080		
0.2	0.109 ± 0.060	0.102 ± 0.032	0.105 ± 0.018
0.13	0.092 ± 0.050		
0.1	0.094 ± 0.061		

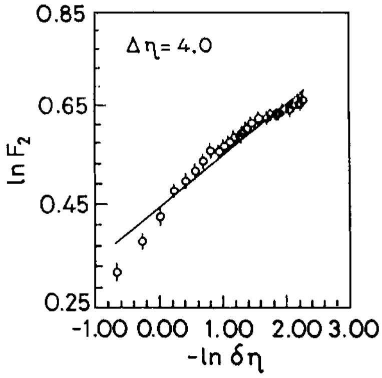

Fig. 4. Plot of $\ln F_2$ against $-\ln \delta\eta$

dependence of FCs, are present as essential contents in our data and Fig. 2(a–b) thus speak in favour of the α-model. Similar observations have also been reported by the NA22 Collaboration [14] for π^+p and k^+p interaction at 250A GeV and EMC Collaboration [15] for muon-proton interaction at 280 GeV. The exponent ϕ_{ij} are expected to follow the simple relation [1]:

$$\phi_{ij} = \varphi_{i+j} - \phi_i - \phi_j = ij\phi_2, \tag{7}$$

where the first equality sign is due to the α model and the second to the log-normal approximation [16]. The second part actually connects all ϕ_{ij}'s to the lowest order (order two) intermittency exponent which can be determined with least amount of error for any experimental data set. The log normal approximation suggests that ϕ_{ij} should rise linearly with the product $(i \times j)$ and the slope of the curves (ϕ_{ij} *vs.* $i \times j$) should be equal to the second order intermittency exponent. This is verified by plotting ϕ_{ij} as a function of the product $(i \times j)$ for different bin widths in Fig. 3(a–d). It is observed from Table 2 that the value of intermittency exponent of the second order (ϕ_2) and the slopes of the curves (Fig. 3(a–d)) i.e. $\Delta\phi_{ij}/\Delta(i \times j)$ are in good agreement with each other reflecting the validity of log normal approximation in our data set. ϕ_2 value is obtained from Fig. 4 by the best fit of $\ln F_2$ against $-\ln \delta\eta$. The F_2 values have been evaluated using the (2).

It has been correctly pointed out by Ochs *et al.* [17, 18] that the real event occurs in three dimensional phase space and its one dimensional projection may reduce or even obliterate the sign of intermittent behaviour. It is not possible to confirm if a system has an intermittent behaviour by probing into one dimensional space only. The "intermittency"-"nonintermittency" ambiguity problem has fortunately been resolved successfully by Seixas

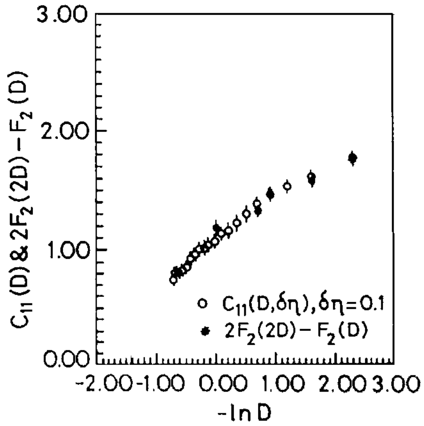

Fig. 5. Verification of the scaling law (10)

[19] who found a dimension-independent relation that can be used as an appropriate tool for identifying intermittent pattern of fluctuation.

Now both F_{11} and F_2 may be represented as [19, 20]

$$F_{11}(D) = \frac{\int_0^\delta \mathrm{d}\eta_1 \int_D^{\delta+D} \mathrm{d}\eta_2 \rho_2(\eta_1, \eta_2)}{\left(\int_0^\delta \rho_1(\eta_1)\mathrm{d}\eta_1\right)\left(\int_D^{\delta+D} \rho_1(\eta_2)\mathrm{d}\eta_2\right)} \tag{8}$$

$$F_2(\delta) = \frac{\int_0^\delta \mathrm{d}\eta_1 \int_0^\delta \mathrm{d}\eta_2 \rho_2(\eta_1, \eta_2)}{\left[\int_0^\delta \rho_1(\eta_1)\mathrm{d}\eta_1\right]^2} \tag{9}$$

are integrals over the two particle correlation function $\rho_2(\eta_1, \eta_2)$, the region of integration being different and δ is the bin width as defined earlier. For F_{11}, correlation function is integrated over two regions of size δ separated by D. If we project any three dimensional process to one dimension, it will have the same consequences on both F_{11} and F_2. So any relation connecting F_{11} and F_2 will have nothing to do with the dimension at which the relation is framed. Seixas [19] worked out the simple dimension-independent relation of the form

$$F_{11}(D) = 2F_2(\delta = 2D) - F_2(\delta = D).$$

This can also be written as

$$C_{11}(D) = 2F_2(\delta = 2D) - F_2(\delta = D). \tag{10}$$

It may be mention that generally C_{ij} is the function of D and δ. The scaling law holds for a particular value of δ. So here C_{ij} is the function of D only. In the light of the scaling relation (10), we present our data in Fig. 5. While the open circles indicate the experimental C_{11} values, the stars the calculated ones using (10). The consistency of the data with the scaling behaviour and conformity with the results obtained in NA22 experiment [14] are well vindicated by the figure.

In conclusion, we observe: (i) Factorial correlators show a power like dependence on the separation distance D for $D < 1$, as expected from the α-model and are independent of bin width when D is small ($D < 1$). This however should not be considered as upholding the uniqueness of α-model as other models with short-range order also make similar predictions. (ii) Validity of the dimension independent scaling relation (10) between factorial moments and correlators confirms the intermittent nature of particle production of our data sample.

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